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B.M.S COLLEGE FOR WOMEN AUTONOMOUS

BENGALURU -560004

SEMESTER END EXAMINATION – APRIL/ MAY- 2023

M.Sc. MATHEMATICS – I Semester

ALGEBRA – I

Course Code: MM101T **QP Code: 11001**

Duration: 3 Hours **Max Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.
2) Answer **any five** full questions.

1. (a) Let $\phi: G \rightarrow G'$ be a homomorphism with Kernel K and let $\bar{N}$ be normal subgroup of $\bar{G}$. Then prove that $\frac{G/K}{N/K} \cong \frac{G}{N}$.
(b) Prove that $I(G) \cong \frac{G}{Z(G)}$ where $I(G)$ is a group of inner automorphisms of G and $Z(G)$ is the center of G.
(c) Show that $T : G \rightarrow G$ defined by $T(x) = x^{-1}$ is an automorphism if and only if G is abelian.

(5+5+4)

2. (a) State and prove the orbit stabilizer theorem.
(b) Derive class equation for finite groups.
(c) By using generator -relator form of S_3, verify the class equation of S_3, where S_3 is a symmetric group.

(5+5+4)

3. (a) In a finite group G, prove that the number of p-Sylow subgroups, n_p divides the index of any p-Sylow subgroup. Further, show that
$$n_p \equiv 1 \ (mod \ p).$$
(b) If G is a group of order $11^2 X\ 13^2$, show that G is simple and abelian.
(c) Find all Sylow subgroups of S_3.

(6+5+3)

4. (a) State and prove Jordan-Hölder theorem.
(b) Prove that S_3 is solvable but not simple.

(10+4)

5. (a) Define an integral domain. Prove that every field is an integral domain.
(b) Let R be a commutative ring with unity whose ideals are $\{0\}$ and R only. Prove that R is a field.
(c) If U is an ideal of a ring R, let $[R: U] = \{x \in R: rx \in U, \forall\, r \in R\}$. Prove that $[R: U]$ is an ideal of R containing U.

(5+5+4)

6. (a) Let R be an integral domain with ideal P then prove that P is a ideal if and only if R/P is an integral domain.
(b) Define Principal Ideal ring. Prove that all fields are Principal ideal rings.
(c) Prove that in a commutative ring with unity a maximal ideal is always a prime ideal.

(4+5+5)

7. (a) Show that every field is an Euclidean ring.
(b) If p is a prime of the form $4n + 1$ then show that $x^2 \equiv -1 \ (mod\ p)$ has a solution.
(c) State and prove unique factorization theorem.

(4+5+5)

8. (a) Prove that $F[x]$ is a principal ideal ring where F is a field.
(b) Define a primitive polynomial. Prove that the product of two primitive polynomials is primitive.
(c) State and Prove Gauss Lemma.

(5+5+4)

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